SRDE: An Improved Differential Evolution Based on Stochastic Ranking

Jinchao Liu Technical University of Denmark Nils Koppels Alle Kgs. Lyngby 2800, Denmark +45 4525 5602

jliu@man.dtu.dk

Zhun Fan Technical University of Denmark Nils Koppels Alle Kgs. Lyngby 2800, Denmark +45 4525 6271

zf@mek.dtu.dk

Erik Goodman 2120 Engineering Building, MSU East Lansing, MI, 48824, USA +01 517 355 6453

goodman@egr.msu.edu

ABSTRACT

In this paper, we propose a methodology to improve the performance of the standard Differential Evolution (DE) in constraint optimization applications, in terms of accelerating its search speed, and improving the success rate. One critical mechanism embedded in the approach is applying Stochastic Ranking (SR) to rank the whole population of individuals with both objective value and constraint violation to be compared. The ranked population is then in a better shape to provide useful information e.g. direction to guide the search process. The performance of the proposed approach, which we call SRDE (Stochastic Ranking based Differential Evolution) is investigated and compared with standard DE with two variants of mutation strategies. The experimental results show that SRDE outperforms, or at least is comparable with standard DE in both variants in all the tested benchmark functions.

Categories and Subject Descriptors

I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search – *Heuristic methods*.

General Terms

Algorithms.

Keywords

Differential evolution, stochastic ranking, constrained optimization

1. INTRODUCTION

Since it advent [1], DE has been considered a powerful constrained optimization tool. It has been applied in a large variety of engineering optimization applications [2]. Many researchers have also proposed different ways to improve the performance of standard DE, in terms of its search speed and successful rate.

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Noman and Iba[7] proposed a hybrid DE which adopted an adaptive local search algorithm to accelerating DE. Combining several strategies, Qin and Suganthan[13] proposed a self-adaptive DE(SaDE), SaDE chooses different strategies and adjusts the control parameters of DE according to previous search information. Sun et al.[14] proposed EDA that combines the standard DE with the estimation of distribution algorithm. EDA has been used to locate the most promising area to guide the search. Rahnamayan et al [6] proposed OBL which employs opposition-based learning to initialize the population and for the population to generate jumping in each generation to accelerate the convergence speed of DE.

1. Initialize

Generate the initial generation P_0 , t=1;

- 2. While termination criteria not satisfied do:
- 3. Evaluate population P_t .
- 4. For each individual x^t in P_t
- 5. Generate a trial vector v^t by mutation and crossover
- 6. If v^t is better than x^t then 7. $x^t \leftarrow v^t$; 8. End if 9. End Foreach 10. $t \leftarrow t+1$; 11. End while

Fig. 1. Pseudo-code of standard DE

This paper proposes a simple, but very useful mechanism that can accelerate the search speed of standard DE. From empirical study, it is also shown that it can increase the successful rate. In this approach, stochastic Ranking [3] is first applied to rank the whole population, with both objective value and level of constraint violation as comparison criteria. The ranked population is then able to provide valuable directional information to guide the search of DE. The resulting algorithm, which we call SRDE (Stochastic ranking based Differential Evolution) can consistently outperforms, or at least is comparable to standard DE in a comprehensive test with 24 benchmark problems [15].The remainder of the paper is organized as follows. Section 2 briefly reviews standard DE. Section 3 explains the idea and algorithm flow of SRDE. Section 4 presents experimental verification. Finally, the work is concluded in section 5.

2. DIFFERENTIAL EVOLUTION 2.1 Standard DE

As a population -based evolutionary algorithm, DE has been used to solve a large number of real-parameter optimization problems. Unlike other Evolutionary Algorithms (EAs), DE employs difference of individuals in generation of new individuals. In each iteration, a trial vector is generated by combining a parental individual and the difference vector(s) of several other individuals in the same population. If this trial vector is better than the parental individual, it will become a new offspring and go into the next generation.

Fig. 1 shows the pseudo code of the standard DE. As can be seen, DE first creates an initial population distributed randomly over the whole search space. Then, in each generation, DE creates a trial vector for each individual according to a certain mutation strategy. A crossover operation is then executed, followed by a selection operation, in which DE applies a *Knock-out* competition to select the survivor from the parental individual and its corresponding trial vector. The winner, then, goes into the new generation. This scheme implicitly implements 'elite reserve' model, which typically is very important for optimization.

Many variants of standard DE have been proposed, including the following five different mutation strategies:

"rand/1":
$$\vec{v}_i^T = \vec{x}_{r1}^T + F(\vec{x}_{r2}^T - \vec{x}_{r3}^T)$$

"best/1": $\vec{v}_i^T = \vec{x}_{best}^T + F(\vec{x}_{r1}^T - \vec{x}_{r2}^T)$
"current to best/1":
 $\vec{v}_i^T = \vec{x}_i^T + F(\vec{x}_{best}^T - \vec{x}_i^T) + F(\vec{x}_{r1}^T - \vec{x}_{r2}^T)$
"rand/2":
 $\vec{v}_i^T = \vec{x}_{r1}^T + F(\vec{x}_{r2}^T - \vec{x}_{r3}^T) + F(\vec{x}_{r4}^T - \vec{x}_{r5}^T)$
"best/2":
 $\vec{v}_i^T = \vec{x}_{best}^T + F(\vec{x}_{r1}^T - \vec{x}_{r2}^T) + F(\vec{x}_{r3}^T - \vec{x}_{r4}^T)$

where the indexes r_1 , r_2 , r_3 , r_4 and r_5 represent five different integers generated within range [1, Np] at random, which should also not be equal to $i \cdot \vec{x}_{best}^T$ denotes the 'best' individual at generation T.

2.2 Different Rules of Handling Constraints

Very few constraint handling techniques have been reported in differential evolution for constrained optimization applications. Two very important and similar techniques are proposed by Lampinen [16] and Becerra and Coello [17]. Both techniques use three rules for the replacement during the selection procedure, and first two are the same. They are:

- A feasible individual is always better than an infeasible individual.
- If both individuals are feasible, the one with better

value of the objective function is selected for the next generation.

The third rule, regarding the situation when both individuals are infeasible, is different. In Lampinen's approach, the comparison is made in the Pareto sense in the constraint violation space. It can be expressed as:

 If both individuals are infeasible, the parent is replaced if the new individual has lower or equal violation for all the constraints.

In Becerra and Coello's approach, a sum of normalized constraint violation is used for comparison, and can be written as:

 If both individuals are infeasible, the individual with less level of constraint violations is better. The level of constraint violation is measured with normalized constraints with the expression of

$$viol(x_j) = \sum_{c=1}^{constr} \frac{g_c(x)}{g_{max\,c}}$$
, where $g_c(x)$ are the

violated constraints of the problem, and $g_{\max c}$ the

largest violation of the constraint $g_{c}(x)$ found so far.

It is worthwhile to point out that both approaches bear some resemblance with an approach proposed by Deb [18] previously, even though Deb's approach is not based in differential evolution. The key difference also lies in the comparison for the case of two infeasible individuals: Lampinen's method makes the comparison in the Pareto sense, Deb' method sums all the constraint violations and compares a single value, Becerra and Coello's method makes normalization for the constraints violations before summing them together.

3. SRDE: STOCHASTIC RANKING BASED DIFFERENTIAL EVOLUTION 3.1SRDE/rand/1

The 'rand/1' mutation strategy used in standard DE provides no information of direction towards the global optimum. If the information of direction can be obtained and utilized in the search process, the performance of the algorithm has a potential to be improved. To avoid the search to be stuck in local minimum, however, the direction information should not be local, but global. To define a 'global direction' information for the whole population is not an easy task, especially when each individual has actually two features to compare with others in a constraint optimization problem – one feature is objective value, the other is level of constraint violation. How to optimally balance them in the comparison procedure presents a challenge.

Stochastic Ranking (SR) [3] provides a convenient and powerful mechanism to balance the dominance in ranking the whole population with both objective value and constraint violation as comparison criteria. The pseudo code of SR is provided in Fig. 3.

The improved DE algorithm, SRDE is designed with a focus on a modified mutation strategy, which can be described in more details as the following: for generation of trial vectors, the whole population is first made to undergo a stochastic ranking procedure. Then the ranked population is divided into two parts –

1. Initialize:

| 2. | Parameters: N_P , F , p_f , p_{CR} , γ ; where N_P denotes the size of population; F denotes scaling factor; p_f is a parameter |
|-----|--|
| | used in stochastic ranking. p_{CR} denotes the probability of crossover, γ represents the number of individuals in the upper part of the |
| | population \mathcal{Q}_{l} . |
| 3. | Generate the initial generation P_0 , $t=0$; |
| 4. | While termination criteria not satisfied do: |
| 5. | Evaluate population $P_t:(f, \varphi) = eval(P_t)$; where f, φ denote objective and violation of constraints, respectively. |
| 6. | Rank population using stochastic ranking: $I = stochastic _rank(f, \phi, p_f)$; |
| 7. | Divide population into two sets: |
| 8. | $Q_{1}^{t} = \left\{ x_{I(1)}^{t} x_{I(2)}^{t} \dots x_{I(\gamma)}^{t} \right\};$ |
| 9. | $Q_2^t = \left\{ x_{I(\gamma+1)}^t x_{I(\gamma+2)}^t \dots x_{I(\lambda)}^t \right\};$ |
| 10. | For $k = 1$ to N_P do |
| 11. | Select $x_{r_1}^t \in P_t$, $x_{r_2}^t \in Q_1^t$, $x_{r_3}^t \in Q_2^t$ at random; |
| 12. | $u_k^t \leftarrow x_{r_1}^t + F \times \left(x_{r_2}^t - x_{r_3}^t\right);$ |
| 13. | $v_k^t \leftarrow crossover(u_k^t, x_k^t, P_{CR});$ |
| 14. | If V_k^t is better than x_k then $x_k^t \leftarrow V_k^t$; |
| 15. | End if |
| 16. | End for |
| 17. | $t \leftarrow t + 1;$ |
| 18. | End white |
| 1. | Initialize: |
| 2 | Parameters: N_{p} , F , p_{c} , p_{cp} , where N_{p} denotes the size of population: F denotes the scaling factor: p_{c} , is a parameter |
| | r_{p} = r_{p} , r_{f} , p_{CR} , r_{r} , p_{CR} , r_{r} , p_{CR} , r_{r} , p_{T} = r_{r} , r_{f} , $r_{$ |

used in stochastic ranking. p_{CR} denotes the probability of crossover.

- 3. Generate the initial generation P_0 , t = 0;
- 4. While termination criteria not satisfied do:
- 5. Evaluate population $P_t: (f, \varphi) = eval(P_t)$; where f, φ denote objective and violation of constraints, respectively.
- 6. Rank population using stochastic ranking: $I = stochastic _rank(f, \varphi, p_f);$
- 7. For k = 1 to N_P do

8.

Select $x_{r_1}^t = x_{l(1)}^t$ /*the 'best' individual in current population in Stochastic Ranking sense.*/

9. Select $x_{r_2}^t$, $x_{r_3}^t \in P_t$ at random;

10.
$$u_k^t \leftarrow x_{r_1}^t + F \times \left(x_{r_2}^t - x_{r_3}^t\right);$$

11.
$$v_k^t \leftarrow crossover(u_k^t, x_k^t, P_{CR});$$

12. If v_k^t is better than x_k then $x_k^t \leftarrow v_k^t;$

- 13.End if14.End for15. $t \leftarrow t+1$;
- 16. End while

Fig. 2. Pseudo-code of iterative search procedure of SRDE/best/1/bin

upper part and lower part. The upper part comprises of the 'better' individuals who have been ranked high after stochastic ranking procedure. For each individual trial vector, the base individual is selected randomly from the whole population, and the second, third individuals are selected from the upper part and lower part, respectively. The three individuals then make a mutation operation according to 'rand/1' strategy, with the difference vector obtained through extracting one 'good' individual with the 'less-good' individual. It is notable that in this way the difference vector will always be directed towards the upper part of the population, thus leading the population to search upwards (Fig. 4). This procedure is repeated until the whole population of trail vectors is obtained. The rest of the algorithm is the same as standard DE/rand/1. The overall procedure of the SRDE/rand/1 algorithm can be illustrated using the pseudo-code listed in Fig. 2.

1.
$$I_j = j$$
, $\forall j \in \{1, ..., \lambda\}$
2. for $i = 1$ to N do
3. for $j = 1$ to $\lambda - 1$ do
4. sample $\mu \in U(0, 1)$
5. if $\varphi(I_j) = \varphi(I_{j+1}) = 0$ or $(\mu < P_f)$ then
6. if $f(I_j) > f(I_{j+1})$ then
7. swap (I_j, I_{j+1}) ;
8. end if
9. else
10. if $\varphi(I_j) > \varphi(I_{j+1})$ then
11. swap (I_j, I_{j+1}) ;
12. end if
13. end if
14. end if
15. end for
16. if no swap done break
17. end for
16. Fig. 3. Pseudo-code of stochastic ranking [3]

3.2 SRDE/best/1

The mechanism of SR can be embedded not only to rand/1 mutation strategy, but also to other mutation strategies. The resultant variation of SRDE can also achieve performance improvement. In this section, we investigate the embedment of SR to best/1 mutation strategy, thus the SRDE/best/1.

The pseudo code of SRDE/best/1 is show in Fig. 2. In this variation of SRDE, the base individual is always selected as the uppermost individual of the population after SR, therefore the 'best' individual of the population in the SR sense.

4. EXPERIMENT

In this section, two sets of experiments were conducted to compare the performances of DE and SRDE with two different mutation strategies, in the well-known 24 benchmark test problems [15]. The codes were implemented in MATLAB and run on an Intel Core2 laptop with 4G RAM under WINDOWS-XP platform. For each test problem, 50 independent runs were conducted.



Fig. 4. Illustration of the modified mutation strategy in SRDE/rand/1. Note that the population ranked by SR is divided into upper part Q1 and lower part Q2. Difference of one randomly selected individual r2 from Q1 and one randomly selected individual r3 from Q2 form a differential vector pointing towards r2.

4.1 Configuration and Parameters Setting

For all test problems, we set the same values for the parameters used both for standard DE and SRDE, and they are listed as following:

> Population size: 100 [4][6] Maximum number of generation: 10000 Differential factor: F = 0.7 [9] Differential crossover probability: $p_{CR} = 0.8$ Value to reach, VTR = 10⁻⁴ [13]

The additional parameter for Stochastic Ranking: $p_f = 0.45$ [3]

The additional parameters for SRDE/rand/1 are: $\gamma = 0.3$

In this paper, the standard *binary* crossover operator was adopted for all algorithms.

4.2 Comparison of DE and SRDE with rand/1/bin mutation

The purpose of the conducted experiments in this subsection is to compare the performance of standard DE and SRDE with rand/1/bin strategy. In this experiment, median number of solution candidates (NSC) and successful rate (SuR) of 50 runs have been used as performance measures. For convenience of comparison, we further defined two metrics:

$$SuR_{diff} = SuR_{SRDE} - SuR_{DE}$$

$$NSC_{ratio} = \frac{NSC_{DE}}{NSC_{SRDE}}$$

The second metric is the same as Acceleration Rate (AR) defined in [6], which provides a very convenient way to compare search speed. From the results listed in Table 1, it can be seen DE failed in problems g03,g13,g20,g22,g23, and had a very low successful rate in problems g17(20%). For these problems, SRDE had significant improvements, i.e. for g13, SuR is improved from 0% to 44%,



Fig. 5. SuR vs. Problems



Fig. 6. NSC_{ratio} vs. Problems.

for g17, from 20% to 82%, for g23, from 0% to 92%. In all other test problems, the SuRs of DE and SRDE are comparable, except for g21 there is a slightly reduction from 76% to 64%. Fig. 5 and Fig. 6 show the SuRs and NSC_{ratio} of DE and SRDE in all benchmark functions respectively. The horizontal axis of Fig. 5 and Fig. 6 represents the new index of test problems according to the difficulty level of problems in an ascending order [19]. Table 1 also shows that SRDE outperforms DE in terms of search speed. SRDE had less NSCs than those of DE in all test functions in which they can both succeed to achieve the optimal. For those test functions that either SRDE or DE cannot find optimal, no information of NSCs will be used for comparison. Fig. 6 also demonstrates this in a graphical way, in which the vertical axis is the NSC_{ratio} As we explained before, NSC_{ratio} provides a very good metric to compare the search speed. From the average of NSC_{ratio} value of all test problems, it can be concluded that the average improvement of convergence speed was 134.4%. Fig. 9 show the comparison of convergence speed of SRDE/rand/1/bin and DE/rand/1/bin in test function 1, 2, 15, 18.

4.3 Comparison of DE and SRDE with best/1/bin mutation strategy

In this subsection, comparison study of DE and SRDE with respect to best/1/bin strategy has been carried out. The results are showed in Table 2. It can be seen from Table 2 that SRDE improved the successful rate in problem g13, g17, g21 considerably while in the rest problems the results are comparable. In terms of the convergence speed, it can be seen that SRDE could obtain the optimal with much less NSCs than DE in problems g05, g11, g13, g14, g15, g17, g23, although in some problems SRDE needed slightly more NSCs. It is noted that to avoid one value to dominate others in calculating the average improvement of convergence speed, we discount the *NSC_{ratio}* of g13, which is 14.96 and significantly bigger than others. The average improvement of convergence speed is therefore 82.15%.

Fig. 7 and Fig. 8 also show the improvement of success rate and convergence speed of SRDE over DE with best/1/bin strategy. It is notable that the horizontal axis of Fig. 7 and Fig. 8 also represents the new index of test problems according to the difficulty level of problems in an ascending order as in Fig. 5 and Fig. 6. Fig. 10 show the comparison of convergence speed of SRDE/best/1/bin and DE/best/1/bin in test function 5, 11, 13, 15.







5. CONCLUSION

In this paper, we propose a new scheme of DE, SRDE to improve the performance of the standard DE in constraint optimization applications. One critical mechanism embedded in SRDE is applying SR to rank the whole population of individuals with both objective value and constraint violation to be compared, before evolutionary operations are used. The ranked population is then able to provide useful information e.g. direction in the mutation operation to guide the search process. Li [5] also takes advantage of directional information within DE framework for multiobjective optimization. However, the way of extracting directional information is quite different from ours. The comprehensive experimental results show that SRDE outperforms, or at least is comparable with standard DE using both rand/1/bin and best/1/bin mutation strategies in all 24 tested benchmark functions, in terms of both convergence speed and success rate. The convergence speed, however, is improved much more significantly. For the rand/1/bin mutation strategy, the improvement ratio is 134.4%, for best/1/bin strategy, the ratio is 82.15%. It has also been shown in another application work [20] that a slight variation of SRDE can achieve much better optimization solution than standard DE and some other state-ofthe-art EAs in a MEMS design optimization problem.

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| DE(rand/1/bin) | | | | SRDE(rand/1/bin) | | | | 1 | |
|----------------|------------------------|----------------------|----------------------|------------------|------------------------|----------------------|----------------------|--------------|---------------|
| Prob. | Median of NSC(x100) | Feasible Rate(FR) | Success Rate(SuR) | Prob. | Median of NSC(x100) | Feasible Rate(FR) | Success Rate(SuR) | SuR_{diff} | NSC_{ratio} |
| g01 | 607 | 100% | 100% | g01 | 334 | 100% | 100% | 0 | 1.8175 |
| g02 | 7208 | 100% | 100% | g02 | 2036 | 100% | 100% | 0 | 3.5403 |
| g03 | - | 100% | - | g03 | - | 100% | - | - | - |
| g04 | 412 | 100% | 100% | g04 | 295 | 100% | 100% | 0 | 1.3966 |
| g05 | 3377 | 100% | 100% | g05 | 442 | 100% | 100% | 0 | 7.6403 |
| g06 | 223 | 100% | 100% | g06 | 179 | 100% | 100% | 0 | 1.2458 |
| g07 | 6473 | 100% | 100% | g07 | 3409 | 100% | 100% | 0 | 1.8988 |
| g08 | 30 | 100% | 100% | g08 | 24 | 100% | 100% | 0 | 1.2500 |
| g09 | 1076 | 100% | 100% | g09 | 769 | 100% | 100% | 0 | 1.3992 |
| g10 | 7514 | 100% | 100% | g10 | 4303 | 100% | 100% | 0 | 1.7462 |
| g11 | 529 | 100% | 100% | g11 | 141 | 100% | 100% | 0 | 3.7518 |
| g12 | 67 | 100% | 100% | g12 | 63 | 100% | 100% | 0 | 1.0635 |
| g13 | - | 100% | - | g13 | 5737 | 100% | 44% | +44% | INF |
| g14 | 5828 | 100% | 100% | g14 | 3114 | 100% | 100% | 0 | 1.8715 |
| g15 | 1337 | 100% | 100% | g15 | 191 | 100% | 100% | 0 | 7.0000 |
| g16 | 283 | 100% | 100% | g16 | 217 | 100% | 100% | 0 | 1.3041 |
| g17 | 4821 | 100% | 20% | g17 | 2574 | 100% | 82% | +62% | 1.8730 |
| g18 | 5956 | 100% | 100% | g18 | 2594 | 100% | 100% | 0 | 2.2961 |
| g19 | 5415 | 100% | 100% | g19 | 5318 | 100% | 100% | 0 | 1.0182 |
| g20 | - | - | - | g20 | - | - | - | 0 | - |
| g21 | 1688 | 100% | 76% | g21 | 1355 | 100% | 64% | -12% | 1.2458 |
| g22 | - | - | - | g22 | - | - | - | 0 | - |
| g23 | - | - | - | g23 | 8967 | 100% | 92% | +92% | INF |
| g24 | 83 | 100% | 100% | g24 | 70 | 100% | 100% | 0 | 1.1857 |
| Ave | | | | | | | | | 2.3443 |

 TABLE 1

 COMPARISON OF DE AND SRDE WITH RAND/1/BIN STRATEGY

TABLE 2

COMPARISON OF DE AND SRDE WITH BEST/1/BIN STRATEGY

| DE(best/1/bin) | | | | SRDE(best/1/bin) | | | | | |
|----------------|-----------|----------|-----------|------------------|-----------|----------|-----------|--------------|---------------|
| Prob. | Median of | Feasible | Success | Prob. | Median of | Feasible | Success | SuR_{diff} | NSC_{ratio} |
| 1100. | NSC(x100) | Rate(FR) | Rate(SuR) | 1100. | NSC(x100) | Rate(FR) | Rate(SuR) | | |
| g01 | 190 | 100% | 70% | g01 | 199 | 100% | 72% | +2% | 0.9548 |
| g02 | - | 100% | - | g02 | - | 100% | - | - | - |
| g03 | - | 100% | - | g03 | - | 100% | - | - | - |
| g04 | 213 | 100% | 100% | g04 | 211 | 100% | 100% | 0 | 1.0095 |
| g05 | 1362 | 100% | 100% | g05 | 194 | 100% | 100% | 0 | 7.0206 |
| g06 | 132 | 100% | 100% | g06 | 124 | 100% | 100% | 0 | 1.0645 |
| g07 | 1436 | 100% | 100% | g07 | 1452 | 100% | 100% | 0 | 0.9890 |
| g08 | 16 | 100% | 100% | g08 | 17 | 100% | 100% | 0 | 0.9412 |
| g09 | 369 | 100% | 100% | g09 | 372 | 100% | 100% | 0 | 0.9919 |
| g10 | 1976 | 100% | 100% | g10 | 1988 | 100% | 100% | 0 | 0.9940 |
| g11 | 121 | 100% | 100% | g11 | 33 | 100% | 100% | 0 | 3.6667 |
| g12 | 29 | 100% | 100% | g12 | 27 | 100% | 100% | 0 | 1.0741 |
| g13 | 3023 | 100% | 40% | g13 | 202 | 100% | 72% | +32% | 14.96 |
| g14 | 1353 | 100% | 100% | g14 | 1048 | 100% | 100% | 0 | 1.2910 |
| g15 | 557 | 100% | 100% | g15 | 85 | 100% | 100% | 0 | 6.5529 |
| g16 | 134 | 100% | 100% | g16 | 133 | 100% | 100% | 0 | 1.0075 |
| g17 | 1897 | 100% | 40% | g17 | 1222 | 100% | 72% | +32% | 1.5524 |
| g18 | 845 | 100% | 80% | g18 | 864 | 100% | 76% | - 4% | 0.9780 |
| g19 | 2000 | 100% | 100% | g19 | 1944 | 100% | 100% | 0 | 1.0288 |
| g20 | - | - | - | g20 | - | - | - | - | - |
| g21 | 780 | 100% | 30% | g21 | 796 | 100% | 52% | +22% | 0.9799 |
| g22 | - | - | - | g22 | - | - | - | - | - |
| g23 | 4538 | 100% | 90% | g23 | 3041 | 100% | 88% | - 2% | 1.4923 |
| g24 | 54 | 100% | 100% | g24 | 53 | 100% | 100% | 0 | 1.0189 |
| Ave | | | | | | | | | 1.8215 |



Fig. 10. Convergence graphs of DE and SRDE with best/1 mutation strategy in problems 5, 11, 13, 15